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RADIATING HYPERVELOCITY COUETTE AND BOUNDARY-LAYER FLOW IN AIR†

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Abstract—An analytical solution for high speed Couette flow of a gray gas is given for the situation in which the dominant physical mechanisms are the production of energy by viscous dissipation and its transfer by radiation. The analysis is extended to air for the non-gray, optically thin case including thermal conduction, and numerical results are presented to show the range of validity of the radiation controlled solution. The Couette flow results are then applied to the problem of the hypervelocity, laminar boundary layer on a flat plate in air. An approximate expression for the Planck mean absorption coefficient of air is used, and the results obtained confirm the hypotheses that these boundary layers will be optically thin and radiation controlled. The analysis results in simple expressions for estimating the boundary-layer temperature and heat-transfer coefficient.

NOMENCLATURE

B,	$\equiv \sigma T^4/\pi$, integrated Planck function;
C_μ, C_k,	constants defined by equation (26);
K,	(linear) absorption coefficient;
$\bar{K}$,	Planck mean absorption coefficient;
k,	thermal conductivity;
L,	thickness of Couette flow layer;
q,	radiative flux;
q^+, q^-,	half-range radiative fluxes;
Re,	$\equiv \rho_e u_e x/\mu_e$, Reynolds number;
T,	temperature;
T_R,	parameter defined by equation (27);
u,	velocity parallel to wall;
x, y,	distances parallel and normal to wall.

Greek symbols

δ,	thickness of boundary layer;
ε,	emissivity of wall;
μ,	viscosity;
ρ, ρ_0,	density; standard density;
τ,	$\equiv \int_0^y K\,dy$, optical depth;
Φ,	rate of dissipation per unit volume;
ϕ,	$\equiv \Phi/K$.

Subscripts

e,	conditions at outer edge of layer.

1. INTRODUCTION

When vehicles returning from interplanetary missions enter the Earth's atmosphere the boundary-layer temperatures may become so high that radiation is the dominant mode of heat transfer. On the other hand, radiation provides a mechanism by which energy may be removed from the boundary layer, and thus tends to reduce the maximum temperatures which the flow will attain.

When radiant energy transfer is included in the conservation equations, the flat plate boundary layer no longer gives rise to a similarity solution except for the limiting case of optically thick radiation. (On the basis of the results of this paper, the optically thick case appears to be physically unrealistic.) This was pointed out by Koh and DeSilva [1] who gave some numerical

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solutions for the optically thin boundary layer in air for flight conditions at which the effects due to radiation turned out to be negligible. A perturbation analysis of the optically thin boundary layer with "small" radiation has been made by Cess [2], and some numerical solutions for a hypothetical gray gas are presented by Oliver and McFadden [3].

In this paper an approximate analysis of the hypervelocity boundary layer on a flat plate in air is obtained by first considering the corresponding Couette flow problem. An analytical solution for the Couette flow of a purely radiating, gray gas which is valid for all optical depths is given in Section 2. The case of optically thin Couette flow in air with combined radiation and thermal conduction is treated in Section 3. The results of these two analyses are applied to the boundary-layer problem in Section 4.

2. COUETTE FLOW MODEL

In this model the boundary layer is replaced by a parallel flow of thickness L bounded by an opaque wall of emissivity ε at $y = 0$ and by a constant temperature region at $y > L$ (Fig. 1). The model is thus equivalent to a high speed Couette flow (i.e. parallel flow with dissipation) bounded by a fixed, opaque wall at $y = 0$ and a moving, transparent wall at $y = L$ through which blackbody radiation corresponding to temperature T_e is transmitted. Some numerical solutions of Couette flow for combined radiation and conduction have been obtained by Viskanta [4] using the exact equation of radiative transfer for a gray gas and a series expansion of the integrated Planck function, and further numerical solutions of this problem for flow between two opaque walls have been given by Greif [5] using an approximate equation of radiative transfer.

When the energy transfer is controlled by radiation, the heat flux approaches that given by the solution for pure radiation and the effects of conduction on the temperature profile are confined to thin layers near the walls. This has been shown for a stationary gas by Viskanta

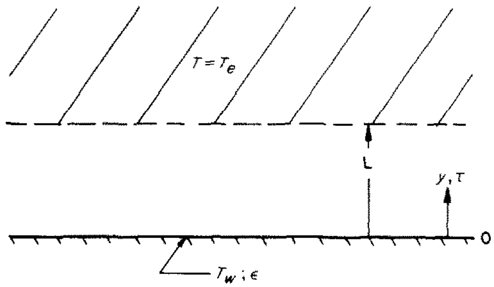

FIG. 1. Schematic drawing of Couette flow model.

and Grosh [6] and by Lick [7], and extended to the case of high speed Couette flow by Greif [5]. In this section, analytical results are obtained for the temperature profiles and heat flux using an approximate equation of radiative transfer and neglecting the effects of heat transfer by conduction.

In the absence of convection and conduction, the energy equation reduces

$$dq/dy = \Phi. \tag{1}$$

Upon assuming that the gas is gray, i.e. $K = K[T(y)]$, the energy equation can be written in terms of optical depth as

$$dq/d\tau = \phi \tag{2}$$

where

$$\phi \equiv \Phi/K = (\mu/K)(du/dy)^2. \tag{2a}$$

It is now assumed that $\phi(\tau) = \text{const}$. This assumption, which is correct for Couette flow with constant fluid properties, may be taken as a crude approximation for high speed flow. Equation (2) can then be immediately integrated to give

$$q(\tau) = \phi\tau + q(0). \tag{3}$$

The differential equation of radiative transfer in one dimension based upon the Milne–Eddington approximation is [8]

$$\frac{d^2q}{d\tau^2} - 3q = 4\pi\frac{dB}{d\tau} \tag{4}$$

and the corresponding half-range radiative flux equations are [8]

$$dq^+/d\tau + (\sqrt{3})\,q^+ = 2\pi B \quad (5a)$$

$$dq^-/d\tau - (\sqrt{3})\,q^- = -2\pi B \quad (5b)$$

where $q \equiv q^+ - q^-$. The boundary condition at the wall, which is assumed to be gray and to emit and reflect diffusely, is

$$q^+(0) = \varepsilon\pi B_w + (1-\varepsilon)\,q^-(0) \quad \text{at} \quad \tau = 0 \quad (6a)$$

and the boundary condition at $y = L$ is

$$q^-(\tau_L) = \pi B_e \quad \text{at} \quad \tau = \tau_L. \quad (6b)$$

Since conduction has been neglected, one must allow for the existence of temperature jumps $B(0) \neq B_w$ and $B(\tau_L) \neq B_e$ at the boundaries.

Substituting equation (3) into (4) and integrating gives

$$B(\tau) = -(3/8\pi)\,\phi\tau^2 - (3/4\pi)\,q(0)\,\tau + B(0) \quad (7)$$

so that for $\phi = \text{const.}$ the temperature distribution $B(\tau)$ is quadratic in τ as was previously shown by Greif [5].

The constants of integration $B(0)$ and $q(0)$ must now be determined. Evaluating equations (3) and (7) at $\tau = \tau_L$ gives

$$q(\tau_L) = \phi\tau_L + q(0) \quad (8)$$

and

$$B(\tau_L) = -(3/8\pi)\,\phi\tau_L^2 - (3/4\pi)\,q(0)\,\tau_L + B(0). \quad (9)$$

Combining equations (5a) and (5b) yields

$$dq/d\tau + (\sqrt{3})(q^+ + q^-) = 4\pi B \quad (10)$$

The boundary condition at $y = 0$ may be manipulated into the form

$$q^-(0) = \pi B_w - q(0)/\varepsilon$$

and substituted into equation (10) to give

$$\phi + (\sqrt{3})\,[1 - (2/\varepsilon)]\,q(0) + 2(\sqrt{3})\,\pi B_w = 4\pi B(0). \quad (11)$$

Similarly, the boundary condition at $y = L$ gives

$$\phi + (\sqrt{3})\,q(\tau_L) + 2(\sqrt{3})\,\pi B_e = 4\pi B(\tau_L). \quad (12)$$

Equations (8, 9, 11, 12) determine $B(0)$, $B(\tau_L)$, $q(0)$ and $q(\tau_L)$. After algebraic manipulation, the solution for $q(0)$ is

$$q(0) = -\left[1 + \frac{(1-\varepsilon)(\sqrt{3})\,\tau_L}{2 + \varepsilon(\sqrt{3})\,\tau_L}\right]\frac{\varepsilon\phi\tau_L}{2} + \frac{2\pi\varepsilon(B_w - B_e)}{2 + \varepsilon(\sqrt{3})\,\tau_L}. \quad (13)$$

For application to a hypervelocity boundary layer on a real wall, both the free-stream and the wall will be relatively cool, and the approximations

$$B_w \ll \phi\tau_L \quad \text{and} \quad B_e \ll \phi\tau_L \quad (14)$$

will be valid; the quantity

$$\phi\tau_L = \int_0^L \Phi\,dy \quad (15)$$

is independent of K and equal to the rate of energy dissipation per unit wall area. In this case (i.e. for cool boundaries) one obtains

$$\frac{q_w}{\phi\tau_L} = -\frac{q(0)}{\phi\tau_L} = \left[1 + \frac{(1-\varepsilon)(\sqrt{3})\,\tau_L}{2 + \varepsilon(\sqrt{3})\,\tau_L}\right]\frac{\varepsilon}{2}, \quad (16)$$

$$\frac{8\pi B(0)}{\phi\tau_L} = \frac{4 + 4(\sqrt{3})\,\tau_L + (2-\varepsilon)\,3\tau_L^2}{(2 + \varepsilon(\sqrt{3})\,\tau_L)\,\tau_L}, \quad (17)$$

and

$$\frac{8\pi B(L)}{\phi\tau_L} = \frac{4 + 4(\sqrt{3})\,\tau_L + \varepsilon 3\tau_L^2}{(2 + \varepsilon(\sqrt{3})\,\tau_L)\,\tau_L}. \quad (18)$$

The effects of wall emissivity and boundary-layer optical thickness on the heat flux absorbed by the wall (as given by equation 16) are plotted in Fig. 2. For a fixed value of ε, the fraction of energy dissipated which is transferred to the wall, $q_w/\phi\tau_L$, varies from $\varepsilon/2$ for $\tau_L = 0$ to 0·5 as $\tau_L \to \infty$.

Representative temperature profiles (in terms of $B \propto T^4$) obtained using equations (7, 16, 17) are shown in Fig. 3. For $\varepsilon = 0$, $q(0) = 0$ and the profiles have a maximum at $\tau = 0$; for $\varepsilon = 1$,

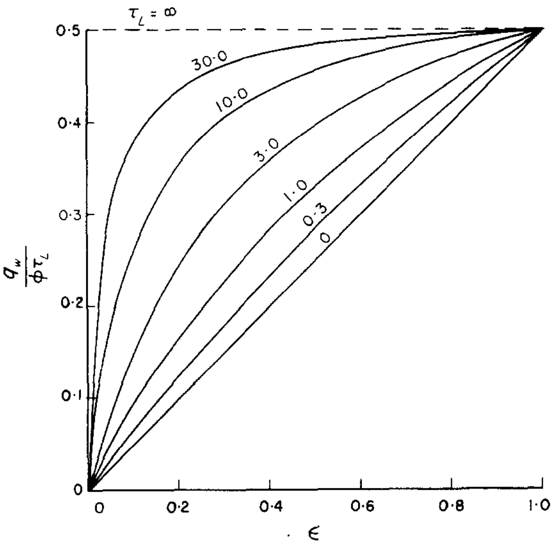

FIG. 2. Variation of heat transfer to the wall with surface emissivity and optical thickness.

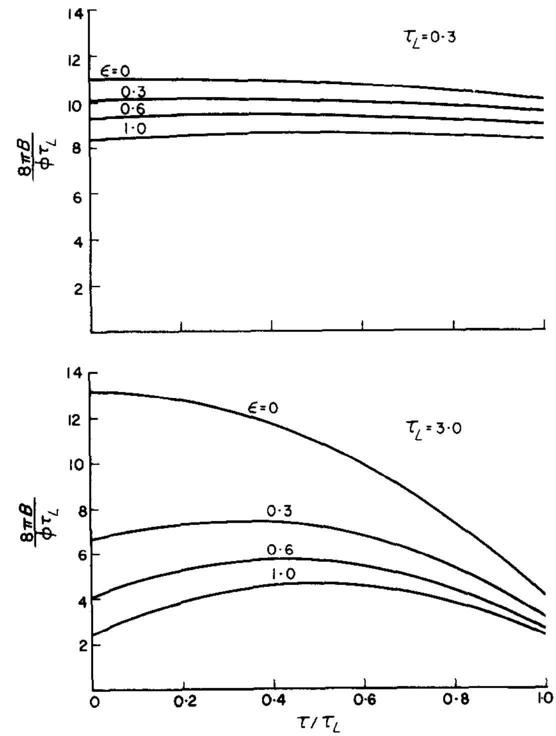

FIG. 3. Typical boundary-layer temperature profiles at two optical thicknesses.

the profiles are symmetrical about $\tau/\tau_L = 0{\cdot}5$. The effect of varying ε on the temperature profiles at a fixed value of τ_L increases as τ_L increases.

For $\tau_L \ll 1$, the temperature is independent of τ and given by

$$8\pi B/\phi = 2 + (2 - \varepsilon)(\sqrt{3})\,\tau_L. \tag{19}$$

For $\varepsilon\tau_L \gg 1$, the boundary temperatures are

$$\frac{8\pi B(0)}{\phi\tau_L} = \left(\frac{2-\varepsilon}{\varepsilon}\right)\sqrt{3} \quad \text{and}$$

$$\frac{8\pi B(L)}{\phi\tau_L} = \sqrt{3}. \tag{20}$$

Thus the pure radiation solution gives rise to temperature jumps at both boundaries for all values of τ_L. In a real gas these temperature discontinuities would, of course, be replaced by thin regions of rapidly varying temperature.

3. OPTICALLY THIN COUETTE FLOW IN AIR

When the layer of gas under consideration is optically thin ($\tau_L \ll 1$), the equation of radiative transfer [equation (4)] reduces to (as shown for example in [9])

$$\mathrm{d}q/\mathrm{d}y = 4\pi B\bar{K} \tag{21}$$

where $\bar{K}$ is the Planck mean absorption coefficient which is equal to K for a gray gas, and given by

$$\bar{K} = (1/B)\int_0^\infty K_\nu B_\nu \,\mathrm{d}\nu$$

for a non-gray gas whose absorption coefficient per unit frequency K_ν varies with frequency ν.

Using equation (21), the energy equation for optically thin Couette flow including thermal conduction of energy is

$$-\frac{\mathrm{d}}{\mathrm{d}y}\left(k\frac{\mathrm{d}T}{\mathrm{d}y}\right) + 4\sigma\bar{K}T^4 = \mu\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2. \tag{22}$$

For air temperatures below 20000°K, Traugott [10] has suggested the use of

$$\bar{K} = 0{\cdot}2\frac{\rho}{\rho_0}\left(\frac{T}{10^4}\right)^5 \text{ cm}^{-1}; \quad [T] = {}^\circ\text{K} \qquad (23)$$

as a simple empirical approximation to the Planck mean absorption coefficient. We assume that the density varies inversely with temperature

$$\rho/\rho_e = (T/T_e)^{-1}, \qquad (24)$$

and the velocity profile is approximated by

$$u/u_e = y/L. \qquad (25)$$

The variation of the transport properties with temperature is approximated by the linear relationships

$$\mu = C_\mu T, \quad k = C_k T. \qquad (26)$$

As nondimensional variables we define

$$\tilde{y} = y/L \quad \text{and} \quad \tilde{T} \equiv T/T_R$$

where

$$T_R \equiv \left[\frac{10^{20}}{0{\cdot}8\sigma}\frac{\rho_0}{\rho_e}\frac{C_\mu u_e^2}{T_e L^2}\right]^{\frac{1}{8}} \qquad (27)$$

would be the (uniform) temperature of the Couette flow layer for the case of pure radiation. Combining equations (23–27) with equation (22) gives

$$-\frac{\alpha}{\tilde{T}}\frac{\mathrm{d}}{\mathrm{d}\tilde{y}}\left(\tilde{T}\frac{\mathrm{d}\tilde{T}}{\mathrm{d}\tilde{y}}\right) + \tilde{T}^7 = 1 \qquad (28)$$

where the conduction–radiation parameter α is defined by

$$\alpha \equiv \frac{C_k T_R}{C_\mu u_e^2}. \qquad (29)$$

The boundary conditions [cf. equation (14)] are approximated by

$$\tilde{T}(0) = 0 \quad \text{and} \quad \tilde{T}(1) = 0. \qquad (30)$$

Solutions of equation (28) with boundary conditions (30) were obtained with an analog computer, and typical profiles (symmetrical about $\tilde{y} = 0{\cdot}5$) are shown in Fig. 4. As $\alpha \to 0$, it can be seen that $\tilde{T}(\tilde{y})$ approaches the pure radiation solution $\tilde{T} = 1$.

For small values of α, the error in the heat transfer to the wall due to neglect of conduction may be estimated as follows. In the case of combined radiation and conduction, the conductive heat flux to the wall is

$$q_C(0) \equiv \left(k\frac{\mathrm{d}T}{\mathrm{d}y}\right)_{y=0} = \frac{C_k T_R^2}{L}\left(\tilde{T}\frac{\mathrm{d}\tilde{T}}{\mathrm{d}\tilde{y}}\right)_{y=0},$$

and in the absence of conduction the radiative heat flux given by equation (16) is

$$q(0) = \frac{\varepsilon\phi\tau_L}{2} = \frac{\varepsilon C_\mu T_R u_e^2}{2L}.$$

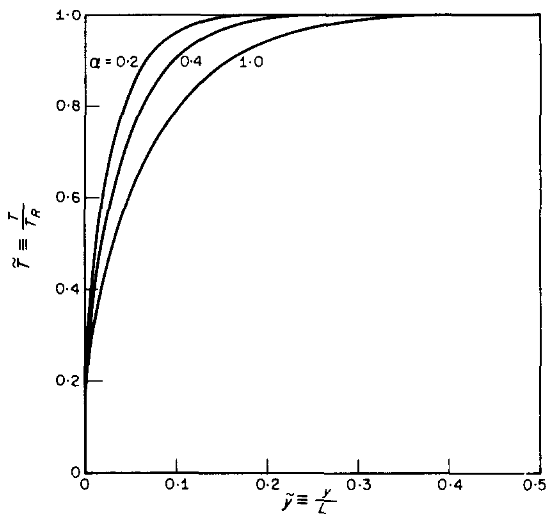

FIG. 4. Effect of conduction–radiation parameter α on the temperature profile in the optically thin case.

Thus the relative effect of conduction is

$$\frac{q_C(0)}{q(0)} = \frac{2\alpha}{\varepsilon}\left(\tilde{T}\frac{\mathrm{d}\tilde{T}}{\mathrm{d}\tilde{y}}\right)_{y=0}. \qquad (31)$$

The computer solutions for $\alpha < 1$ showed that within 2 per cent

$$2[\tilde{T}(\mathrm{d}\tilde{T}/\mathrm{d}\tilde{y})]_{y=0} = 0{\cdot}43/\sqrt{\alpha}$$

so that for the case of optically thin Couette flow of air, the error due to neglecting conduction is

$$q_C(0)/q(0) = 0{\cdot}43(\sqrt{\alpha})/\varepsilon. \qquad (32)$$

4. THE BOUNDARY LAYER IN AIR

In this section we obtain an approximate solution of the boundary-layer problem by using the temperature and velocity profiles of the corresponding Couette flow. We shall assume that the boundary layer is radiation controlled and optically thin, and determine *a posteriori* when these hypotheses are satisfied. Under these conditions the boundary-layer temperature $T(y)$ obtained by setting $\tau_L = 0$ in equation (19) is constant, the fluid properties are constant, and the velocity profile is linear [equation (25)].

The key to obtaining a solution is to relate the arbitrary Couette flow thickness L to the boundary-layer thickness δ. For compressible, laminar, flat plate, boundary-layer flow in a gas satisfying equations (24) and (26), the normal distance y can be expressed in terms of the non-dimensional variable η by [11]

$$y = x(2/Re)^{\frac{1}{2}} \int_0^\eta (T/T_e)\,d\eta^* \qquad (33)$$

where $Re \equiv \rho_e u_e x/\mu_e$ is the free-stream Reynolds number and the velocity distribution $u/u_e \equiv f'(\eta)$ satisfies the Blasius equation. Choosing δ as the value of y for which $u/u_e = 0{\cdot}99$ and using the well known solution of the Blasius equation gives $\eta_\delta = 3{\cdot}5$. Then since $T =$ const., the result of integrating equation (33) is the simple expression

$$\delta = \frac{5T\mu_e^{\frac{1}{2}}x^{\frac{1}{2}}}{T_e\rho_e^{\frac{1}{2}}u_e^{\frac{1}{2}}}. \qquad (34)$$

Combining equations (23–25) and (34) with (19) gives

$$T^9 = 5 \times 10^{18}(\rho_0/\sigma)(u_e^3/x)$$

which, using cgs units, reduces to

$$T = 170\, u_e^{\frac{1}{3}}/x^{\frac{1}{9}}\ ^\circ\mathrm{K}. \qquad (35)$$

It may be noted that this unexpectedly simple, approximate result for the boundary-layer temperature is independent of the free-stream temperature, density, and viscosity.

From equation (16) for $\tau_L \ll 1$, the heat flux to the wall is $q_w = (\varepsilon/2)\,\Phi\delta$. Using equations (25), (26) and (34) gives

$$q_w = \frac{\varepsilon}{10}\left(\frac{\rho_e\mu_e}{x}\right)^{\frac{1}{2}} u_e^{\frac{5}{2}} \qquad (36)$$

which depends upon the result that $T(y)$ is constant but not upon its value. Thus equation (36) is independent of the relationship used for $\bar{K}(\rho, T)$. Combining equation (36) with an appropriate definition of the Stanton number for hypervelocity flow gives another particularly simple result

$$St \equiv q_w/(1/2)\,\rho_e u_e^3 = (\varepsilon/5)Re^{-\frac{1}{2}}. \qquad (37)$$

The analysis which has been presented will be self consistent when the optical depth τ_δ and the conduction–radiation parameter α are small and the temperature restriction ($T < 20000$°K) on the Planck mean absorption coefficient $\bar{K}$ is satisfied. Combining equations (23, 26, 34, 35) and using a value of μ for air at 300°K of $1{\cdot}8 \times 10^{-4}$ g/cm s gives (in cgs units)

$$\tau_\delta \equiv 5 \times 10^{-10}\left(\frac{\rho_e}{\rho_0}\right)^{\frac{1}{2}}\left(\frac{T_e}{300}\right)^{\frac{1}{2}}\frac{u_e^{\frac{7}{6}}}{x^{\frac{1}{18}}}. \qquad (38)$$

Equation (38) shows that, contrary to what one might expect, τ_δ slowly decreases with increasing x. Values of τ_δ are plotted in Fig. 5 over an extended velocity range for a nominal value of $x = 1$ m. These results show that τ_δ will be much less than one for almost any re-entry application. Referring to the Couette flow analysis of Section 2 as illustrated in Figs. 2 and 3, it was seen that values of temperature and heat flux for $\tau_\delta < 0{\cdot}1$ differ negligibly from those for $\tau_\delta = 0$.

The parameters T_R and α were defined in equations (27) and (29), where for our boundary-layer application T_R is given by equation (35). Considering dissociated air as a monatomic perfect gas of average molecular weight $M = 15$, the value of C_k/C_μ is independent of temperature

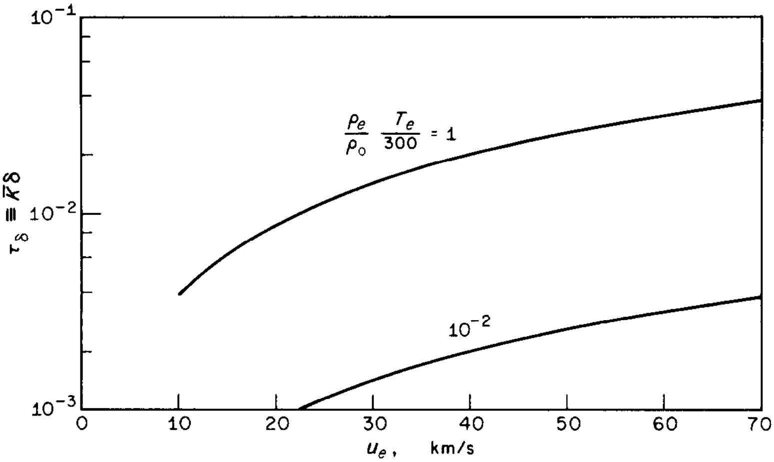

FIG. 5. Optical depth τ_δ based upon Planck mean absorption coefficient for boundary layers in air.

and given by ($Pr \equiv$ Prandtl number)

$$\frac{C_k}{C_\mu} = \frac{k}{\mu} = \frac{c_p}{Pr} = \frac{(5/2)(\mathscr{R}/M)}{2/3}$$
$$= 2{\cdot}1 \times 10^7 \text{ erg/g°K}. \qquad (39)$$

Equation (39) corresponds to the "frozen flow" case since it does not include the possible transfer of energy in the boundary layer due to diffusion of reacting species. Using estimates of Fay [12] for the viscosity and "equilibrium" conductivity k_E of high temperature air would give values of C_{k_E}/C_μ which vary with temperature and density and could give average values of C_{k_E}/C_μ as high as four times that given by equation (39).

Substituting equations (35) and (39) in (29) gives (in cgs units)

$$\alpha = \frac{3{\cdot}6 \times 10^9}{u_e^{\frac{5}{3}} x^{\frac{1}{5}}}. \qquad (40)$$

Values of α are plotted in Fig. 6 which shows that the hypothesis that α is small (i.e. that the boundary layer is radiation controlled) is well satisfied for $u_e > 15$ km/s (say) and marginally satisfied at the lower velocities.

The boundary-layer temperature given by

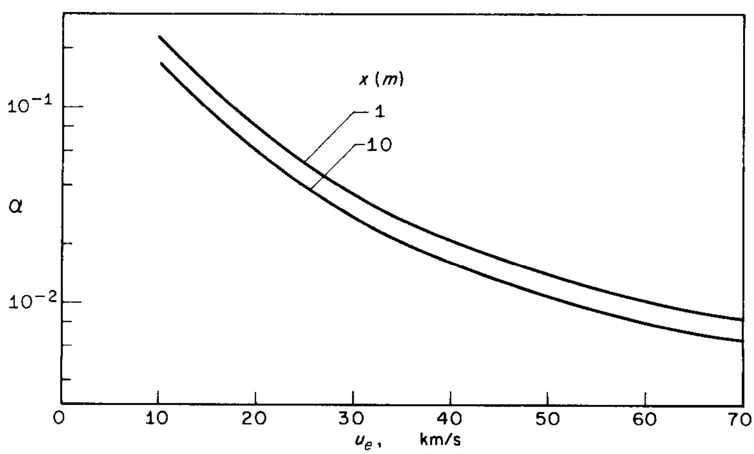

FIG. 6. Conduction–radiation parameter α for boundary layers in air.

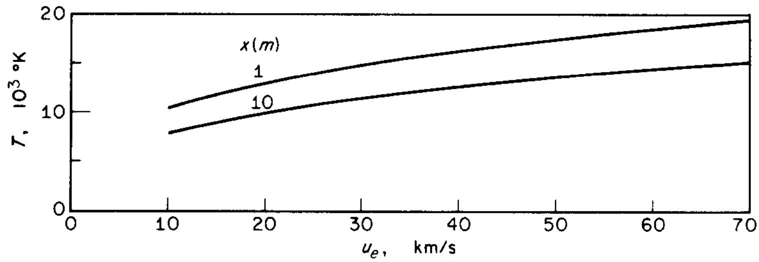

FIG. 7. Temperature in the air boundary layer as function of freestream velocity and distance from leading edge of plate.

equation (35) is plotted in Fig. 7 which shows that the temperature remains below 20000°K. Thus there is a significant range of free-stream conditions for which the analysis is self consistent. For these conditions, the analysis should provide useful estimates of the properties of hypervelocity boundary layers.

REFERENCES

1. J. C. Y. KOH and C. N. DESILVA, Interaction between radiation and convection in the hypersonic boundary layer on a flat plate, *ARS Jl* **32**, 739–743 (1962).
2. R. D. CESS, Radiation effects upon boundary-layer flow of an absorbing gas, *J. Heat Transfer* **86**, 469–475 (1964).
3. C. C. OLIVER and P. W. MCFADDEN, The interaction of radiation and convection in the laminar boundary layer, *J. Heat Transfer* **88**, 205–213 (1966).
4. R. VISKANTA, Heat transfer in Couette flow of a radiating fluid with viscous dissipation, in *Developments in Mechanics*, pp. 376–402. Pergamon Press, Oxford (1965).
5. R. GREIF, Couette flow of a radiating and conducting gas, *AIAA Jl* **3**, 98–102 (1965).
6. R. VISKANTA and R. J. GROSH, Heat transfer by simultaneous conduction and radiation in an absorbing medium, *J. Heat Transfer* **84**, 63–72 (1962).
7. W. LICK, Energy transfer by radiation and conduction, in *Proceedings of the* 1963 *Heat Transfer and Fluid Mechanics Institute*, pp. 14–26. Stanford University Press, Stanford (1963).
8. S. C. TRAUGOTT and K. C. WANG, On differential methods for radiant heat transfer, *Int. J. Heat Mass Transfer* **7**, 269–273 (1964).
9. V. N. ZHIGULEV, YE. A. ROMISHESKII and V. K. VERTUSHKIN, Role of radiation in modern gasdynamics, *AIAA Jl* **1**, 1473–1485 (1963); translated from *J. Engng (U.S.S.R.)* **1**, 60 (1961).
10. S. C. TRAUGOTT, Shock structure in a radiating, heat conducting, and viscous gas, *Physics Fluids* **8**, 834–849 (1965).
11. K. STEWARTSON, *The Theory of Laminar Boundary Layers in Compressible Fluids*, chapter 3. Clarendon Press, Oxford (1964).
12. J. A. FAY, Hypersonic heat transfer in the air laminar boundary layer, in *The High Temperature Aspects of Hypersonic Flow*, pp. 583–605. Pergamon Press, Oxford (1963).

Résumé—Une solution analytique pour un écoulement de Couette à grande vitesse d'un gaz gris est donné dans le cas où les mécanismes physiques dominants sont la production d'énergie par dissipation visqueuse et son transport par rayonnement. L'analyse est étendue à l'air pour les cas non-gris, optiquement mince en tenant compte de la conduction thermique, et l'on présente des résultats numériques pour montrer la gamme de validité de la solution contrôlée par le rayonnement. Les résultats de l'écoulement de Couette sont alors appliqués au problème de la couche limite laminaire à grande vitesse dans de l'air sur une plaque plane. Une expression approchée pour le coefficient d'absorption moyen de Planck de l'air, est employée, et les résultats obtenus confirment l'hypothèse que ces couches limites seront optiquement minces et contrôlées par le rayonnement.

L'analyse aboutit à des expressions simples pour l'estimation de la température de la couche limite et du coefficient de transport de chaleur.

Zusammenfassung—Für die Hochgeschwindigkeits-Couette-Strömung eines grauen Gases wird eine analytische Lösung dafür angegeben, dass die Erzeugung innerer Energie durch Reibung und ihr Transport durch Strahlung die bestimmenden physikalischen Vorgänge darstellen. Die Analyse wird auf Luft ausgedehnt für den nicht-grauen, optisch dünnen Fall einschliesslich Wärmeleitung; um den Gültigkeitsbereich der von der Strahlung bestimmten Lösung zu zeigen, sind numerische Lösungen angegeben. Die Couette-Strömungsergebnisse werden auf das Problem der laminaren Hyperschallgrenzschicht an einer ebenen Platte in Luft angewandt. Ein Näherungsausdruck wird für den mittleren Planck'schen Absorptionskoeffizienten von Luft verwendet und die Ergebnisse bestätigen die Annahme, dass diese Grenzschichten optisch dünn und strahlungsunabhängig sind. Die Analyse liefert einfache Ausdrücke zur Abschätzung der Grenzschichttemperatur und des Wärmeübergangskoeffizienten.

Аннотация—Представлено аналитическое решение высокоскоростного течения Куэтта серого газа, где основным физическим механизмом является образование энергии при вязкой диссипации и её перенос излучением. Анализ распространяется на воздух для случая несерой оптически тонкой среды, включая теплопроводность. Представлены численные результаты, которые показывают область применения решения для случая, когда процесс переноса тепла определяется радиацией. Затем результаты для течения Куэтта применяются к задаче гиперскоростного ламинарного пограничного слоя на плоской пластиние в воздухе. Для планковского среднего коэффициента поглощения в воздухе используется приближенное выражение и полученные результаты подтверждают гипотезу о том, что эти пограничные слои являются оптически тонкими и процесс теплообмена в них определяется радиацией. В результате анализа получены простые выражения для расчета температуры пограничного слоя и коэффициента теплообмена.